Clusters in a magnetic toy model for binary granular piles

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Results on a generalized magnetically controlled ballistic deposition model of granular piles are reported in order to search for the effect of "spin flip" probability q in building a granular pile. Two different regimes of spin cluster site distributions have been identified, a borderline $q_c(\beta J)$ where J is the interaction potential strength.

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I. INTRODUCTION

The physics of granular matter has drawn a great deal of information from percolation theory ideas [1]. Relatively simple models based on analogies have been implemented in order to describe realistic granular pile (static and dynamic) properties [2–7]. Segregation [8], decompaction [9], and avalanches [10] point out to the existence of clusters. However, in describing such materials it is crucial to consider that they are not made of symmetrical entities. It is necessary to introduce at least one degree of freedom for the grain with some coupling to an external field. One can imagine that the degree of freedom, is called a "spin," coupled to a "magnetic field," though the spin can represent any nonmagnetic physical feature of practical interest, like the grain roughness or shape feature. The spin role is to break the spatial isotropic symmetry. The direction of such a spin can represent the position of a grain with respect to neighboring entities, as well as a rotation process. The grain-grain interaction can be imagined to be some (elasticlike) potential containing information on the grain Young, rigidity, bulk modulus, and Poisson ratio, … and geometric aspects [11]. Generalizations to more complex spin models are immediately imagined.

Coniglio and Herrmann presented in Ref. [12] a related view of the granular packing problem and adapted the Ising model and Sherrington-Kirkpatrick spin glass model to granular phenomena obtaining two phase transitions in the system. The short range exchange energy J describing a "spin-spin interaction" is analogous in granular matter to the contact energy between grains. An interpretation of J for flows can be found in Pandey *et al.* [13]. A constrained Ising spin chain has also been recently considered and studied as a toy model for granular compaction [14].

Something which is fully appreciated is the difference in constructing piles in presence or not of vertical walls. A pyramidal pile has not necessarily the same structure, arches, … as a pile in a silo. Moreover, it is quite unrealistic to build a pile from a single grain faucet. Finally, due to its anisotropic shape a grain can rotate during its fall, e.g., in responding to the local wind, or difference in pressure between the upper and lower grain surfaces. Such remarks have motivated us into reexamining a magnetic Tetris-like model (or a magnetic rain) for which the grain is characterized by a spin which can flip during its fall under some energetic condition.

In this paper the role of changing the depositing spin flipping probability during its fall in such a MBD (magnetic ballistic depositon) model is shown to influence the pile density, the pile "magnetization," and the cluster size distribution. In Sec. II, we establish the algorithm rules and briefly comment upon them. In Sec. III we present numerical results for the density and the magnetization of the pile (Secs. IV and V). A critical percolation line is found separating two regimes for the size (mass) distribution of clusters. Finally, in Sec. VI, a brief conclusion can be found.

II. EXPERIMENTAL PROCEDURE

The algorithm for the so called q-MBD model, in contrast to the $\frac{1}{2}$-MBD model [15], goes as follows:

(1) First, we choose a horizontal substrate of spins with a predetermined (for example, antiferromagneticlike or random) configuration; periodic boundary conditions are used.

(2&3x0029; A falling (up or down) spin is dropped along one of the lattice columns from a height $r_{max}+5a$, where r_{max} is the largest distance between an occupied cluster site and the substrate.

(3) At each step down the spin can flip, i.e., change its "sign;" the "up" direction has a probability q.

(4) The spin goes down flipping until it reaches a site perimeter of the cluster at which time the local gain in the Ising energy

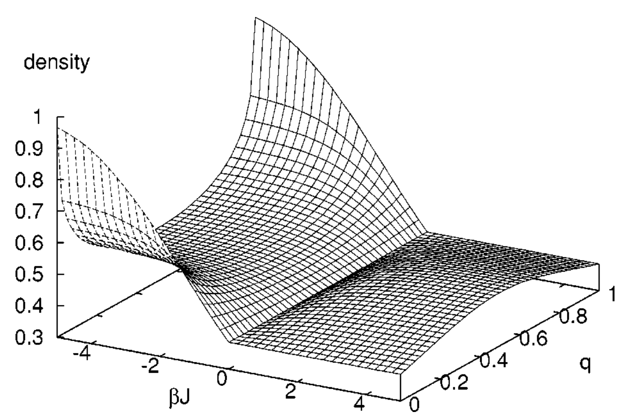

FIG. 1. The dependence of the density on q and βJ.

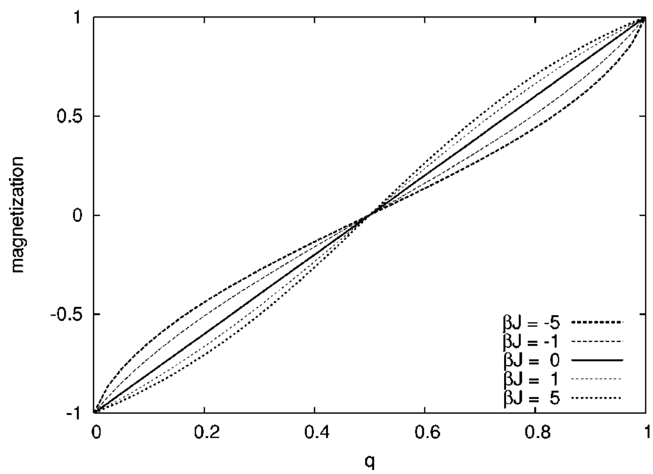

FIG. 2. Dependence of the magnetization on q for several βJ values: F case (dotted lines), AF case (dashed lines).

$$\beta E = -\beta J \sum_{\langle i,j \rangle} \sigma_i \sigma_j \quad (1)$$

is calculated. The fall velocity is irrelevant and there is no backscattering. If the gain is negative the spin sticks to the cluster immediately (sticking probability =1.0) and one goes back to step (2). In the opposite case the spin sticks to the cluster with a rate $\exp(-\Delta\beta E)$ where $\Delta\beta E$ is the local gain in the Ising energy. If the spin does not stick to the cluster it continues going down. Of course, if the site just below the spin is occupied, the spin immediately stops and sticks to the cluster.

(5) After the spin stops one goes back to step (2).

In the $\frac{1}{2}$-MBD model [15] a finite field H and $q=0.5$ were assumed; in the present report we take $H=0$ but enlarge the permissible values of q to $[0,1]$.

III. NUMERICAL RESULTS

All results reported below are for a triangular lattice of horizontal size $L=100$, when the pile made of clusters has reached a 100 lattice unit height, and after averaging over 1000 simulations. The substrate consists of spins with random direction.

A. Density

We define the density of a cluster as ρ=number of spins in the cluster/number of sites on the lattice in which obviously the number of lattice sites in the denominator =10 000.

Figure 1 illustrates the behavior of the density with respect to the q and βJ parameters. This figure convinces us that the results are symmetrical with respect to $q=0.5$. In the $\frac{1}{2}$-MBD model the density varies between 0.38 and 0.47 but in the present q-MBD a spread in density occurs—from almost a completely compact pile in the antiferromagnetic (AF) case and large q to a loosely packed pile in the F case. The lowest density (0.38) occurs for $\beta J=0$ and for border values of q, i.e., 0 and 1, in the F case. For strong enough positive interactions ($\beta J>4$) the density saturates toward the

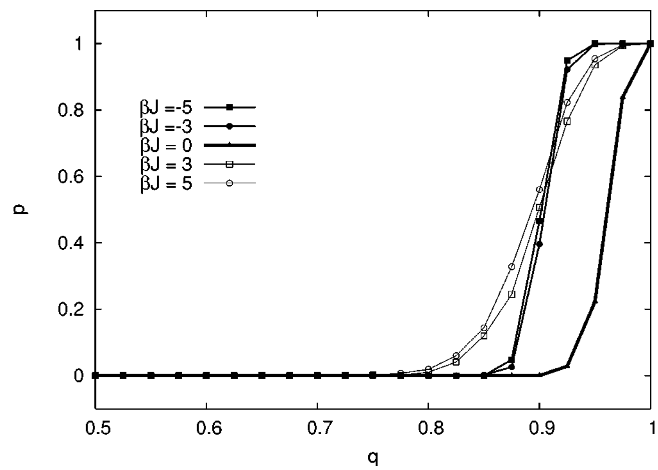

FIG. 3. Dependence of the percolation probability p_c on as a function of q for different βJ values.

value $\rho \approx 0.47$, in the F case. In the AF case, the density varies between 0.38 and 1.0.

B. Magnetization

The dependence of the magnetization defined as

$$M = \frac{n_+ - n_-}{n_+ + n_-} \quad (2)$$

is shown in Fig. 2 as a function of βJ and q, where n_+ and n_- are the number of up and down spins, respectively, i.e., $n_+ = 10\,000\rho_+$. M can be considered as a measure of the difference in grain orientations in the final packing.

The magnetization appears to be the same in the regions ($\beta J>0, q<0.5$) and ($\beta J<0, q>0.5$) (see Fig. 2). Notice that there is no terrace observed here in contrast to the $\frac{1}{2}$-MBD model case [15].

IV. PERCOLATION

Define an up-spin percolating cluster as a cluster of up-spins which extents from the bottom to the top of the sample.

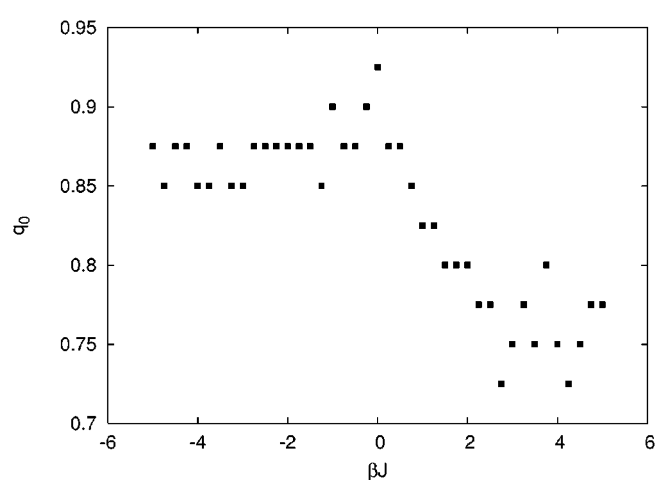

FIG. 4. Dependence of q_0 on βJ.

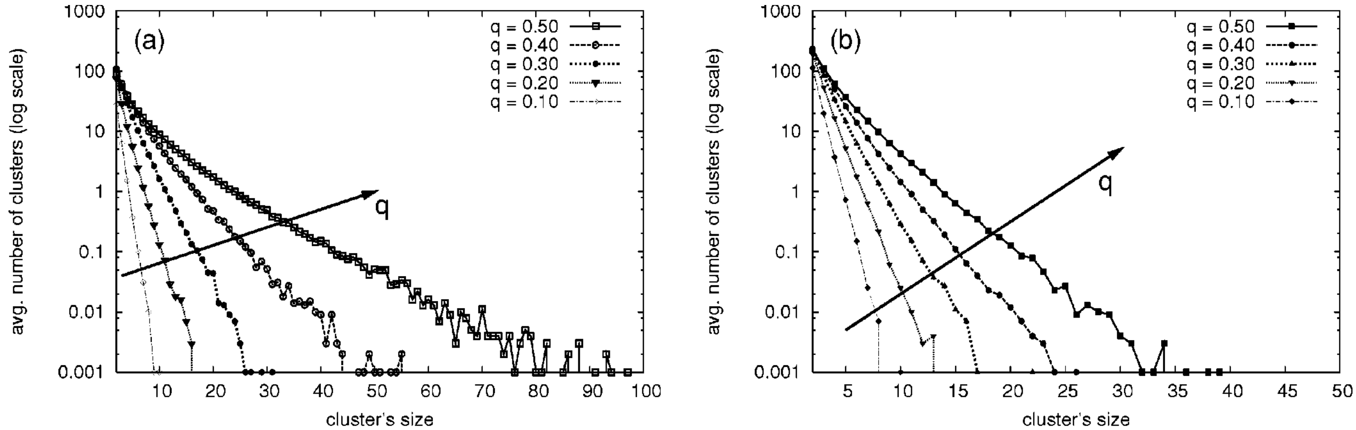

FIG. 5. Examples of the $N_s(s)$ dependence for low values of q on semilog plots: (a) $\beta J=5$; (b) $\beta J=-5$.

At fixed q and βJ the fraction p of piles consisting in such a percolating cluster of up-spins was computed. The behavior of p with respect to q for several values of the βJ parameter is shown in Fig. 3. There is no percolating cluster for $q<0.75$ in the F case and 0.85 in the AF case. The q_0 $[q_0(\beta J):=q$ when $p>0]$ dependence is shown in Fig. 4. Above q_0 when p becomes finite a very fast growth of p is observed, at a given q_c.

The differences between AF and F are understood if one recalls that the density in the AF case is much more sensitive to a change in q than in the F case, for which a change of q induces only a very small variation of the density—recall that the density in this case is $[0.38;0.47]$. On the other hand, a similar variation in q values generates piles with a wide interval of densities in the AF case.

V. THE SIZE (MASS) DISTRIBUTION OF CLUSTERS

The number of clusters with s size is called N_s. Its behavior is shown in Fig. 5 for low q values. Notice that the N_s dependence is approximately a straight line on a semilog plot (all logs are Neperian), i.e.,

$$N_s(s) \propto e^{-k_E s} \quad \text{for} \quad q \approx 0, \tag{3}$$

where k_E is a constant for a fixed $(\beta J,q)$ pair and s is the cluster size (mass). Observe that the existence of a large cluster is more probable in the F case than in the AF case. This is expected when one recalls that the F case favors neighbors with a similar spin direction during the deposition.

For high q values the N_s dependence stops to be an exponential law and becomes a power law (see log-log plots on Fig. 6). Let us postulate

$$N_s(s) \propto s^{-k_P} \quad \text{for} \quad q \approx 1. \tag{4}$$

In order to find the values of q for which the crossover effect occurs, i.e., from an exponential law regime to a power law regime, we have estimated the slopes of the N_s dependencies in both regimes. It seems that one should distinguish pile growth conditions with respect to $q(\beta J)<q_c$ and $q(\beta J)>q_c$.

Analogous to N_s we can define N_h as the number of hole clusters. Unlike the spin clusters there is no difference in N_h dependencies between low and high values of q—all N_h dependencies seem to follow a power law.

VI. CONCLUSIONS

We have presented the extension of the MBD model [15] in order to find the role of the probability of spin flip during deposition (or the degree of freedom modification) in granular downward rainlike flow in two dimensions. The general-

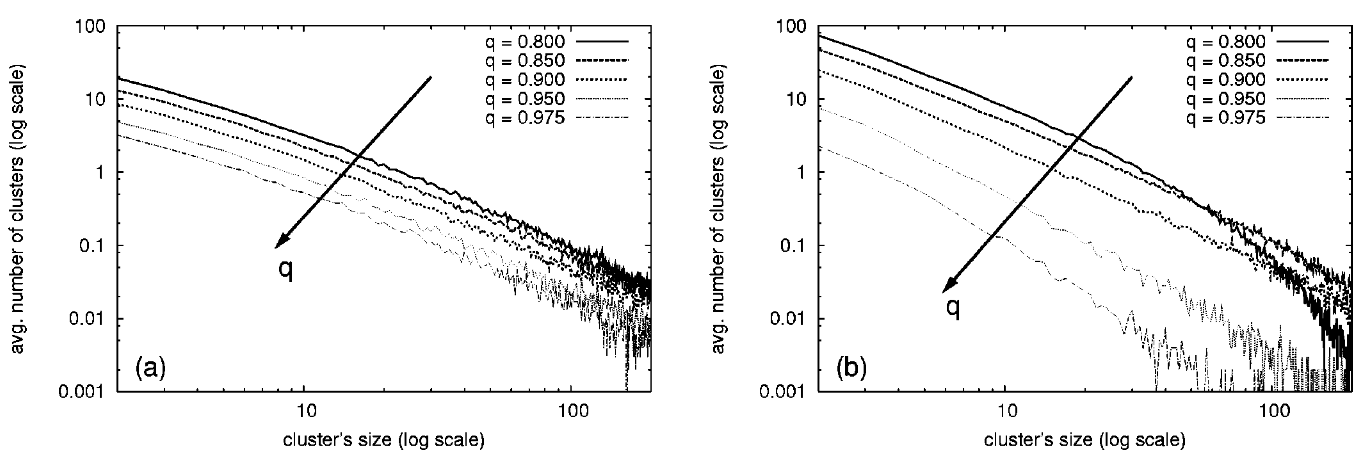

FIG. 6. Examples of the $N_s(s)$ dependence for high values of the q parameter presented on log-log plots; (a) states for ferromagneticlike case, i.e., $\beta J=5$; (b) states for antiferromagneticlike case, i.e., $\beta J=-5$.

ized model, hereby called q-MBD, is a nonequilibrium ballistic deposition model with one degree of freedom. One can imagine that the spins (grains) have different shapes and q can be related to a wind strength such that the grains favor one or another position in order to minimize the pile energy. We have examined the cluster properties through two order parameters, since two characteristic fields (J and q) are intrinsic to the model.

We have investigated the size, or "mass," of the spin clusters created through simulation of the nonequilibrium deposition. The "quenching" of the degree of freedom on the cluster leads to two different regions of spin cluster geometric properties. In the low "field" q region the spin cluster mass distribution follows an exponential law, while in the high q region the distribution is characterized by a power law. The transition between these two regimes is not sharp. The exponential law regime for high strength of interactions (i.e., $|\beta J|>4$) seems to be universal, i.e., independent on the magnitude of the intrinsic parameters but depends only on the sign of βJ, i.e., the characteristic contact interaction potential,—if they are ferromagneticlike or antiferromagneticlike, mechanically repulsive, or attractive.

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